An approach to higher damping capacity: a comparison of material damping with computer-controlled damping

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Abstract

An approach to higher damping by positively utilizing computer-aided control techniques has been investigated by comparing the relative magnitudes of active (computer-controlled) damping and material damping. The active damping effect increases the damping capacity of a brass beam vibration system from $\delta_{\rm H}$ = 0.012 (logarithmic decrement value under free vibration) to $\delta_{\text{max}}=0.3248$ (total maximum damping) and this value of δ_{max} amounts to about 4.5 times the maximum damping observed in the ferromagnetic high damping metal SIA.

I. Introduction

Approaches to antivibration using high damping alloys [1] have been the subject of intensive study because of the possibility of obtaining a system without having to attach any external damping elements such as a dynamic damper [2]. However, there seems to be a practical limitation in such metallurgical approaches owing to the incompatible properties of higher damping and higher stiffness. On the other hand, in recent years active damping [3] by the construction of a computercontrolled system has been developed and successful approaches to antivibration by positively utilizing such control structures have been introduced [4], because such new damping techniques can supply enough dissipation energy by state feedback. When the control signals giving rise to active damping are not saturated, the active damping system is useful practically because of its capability of high damping, unlike passive damping systems such as a dynamic damper or a material damping system where the damping mechanisms are based on internal dissipation inside damping elements. In this paper the active damping effect by the adaptive control scheme AMFC [5] is compared with the high damping metal SIA [6]. There seems to have been no previous comparison between these two damping effects.

2. Construction of active vibration control system

The active vibration control system for a cantilever beam is constructed using the AMFC with variable feedback gain. The main points of the AMFC based on Popov's hyperstability theorem are as follows.

Referring to the block diagram of Fig. 1, let the non-linear multivariable controlled processes be

$$
\dot{x} = A_1(x)x(t) + A_2[x, g(x)] + B(x)u \tag{1}
$$

and let the reference model be

$$
\dot{x}_{\mathsf{m}} = A_{\mathsf{m}} x_{\mathsf{m}}(t) + B_{\mathsf{m}} u_{\mathsf{m}} \tag{2}
$$

The sizes of the various matrices are $x \in \mathbb{R}^n$, $g(x) \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $A_1(x) \in \mathbb{R}^{n \times n}$, $A_2(x) \in \mathbb{R}^{n \times r}$, $B \in \mathbb{R}^{n \times m}$, $A_m \in \mathbb{R}^{n \times n}$, $B_m \in \mathbb{R}^{n \times m}$.

Fig. 1. Block diagram of the adaptive model-following control system based on the hyperstability theorem.

From eqns. (1) and (2) we can establish the adaptive error equation

$$
\dot{e} = A_m e + B_m w \tag{3}
$$

with the non-linear compensation input

$$
w = -C_B(x)u + \psi z \tag{4}
$$

where

$$
\psi = [C_{A_1}(x), \ -C_{A_2}(x), \ z] \tag{5}
$$

The design conditions for the control system to be asymptotically hyperstable are as follows.

First, letting the feedback element be $w = -C_B u + \psi z$, we determine the strictively positive condition for the transfer function $G(s) = C(sI-A_m)^{-1}B_m$. This leads to the choice

$$
C = B_{\rm m}{}^{\rm T} P \tag{6}
$$

where the positive definite matrix *P* is a solution of the Lyapunov equation

$$
A_{\mathbf{m}}^{\mathrm{T}}P + PA_{\mathbf{m}} = -Q \qquad (Q > 0)
$$
 (7)

Then we synthesize the process input signal $u(t)$ so as to satisfy the following integral inequality regarding inputs v and outputs w .

$$
\rho = \int_{0}^{T} v^{T}(-w) d\tau \leq \gamma_0^{2} \qquad \forall t \geq 0
$$
\n(8)

with the result that, since $z = [x_p, g, u_m]^T$,

$$
u = \begin{cases} h_0 ||z|| \text{sgn}(v), & v \neq 0 \\ 0, & v = 0 \end{cases}
$$
 (9)

and

$$
h_0 > h_{\text{oc}} = \frac{\|\psi\|}{\lambda_{\min}[C_B(x)]}
$$
(10)

From control theory [5] it can be said that by taking such a control input $u(t)$, the adaptive tracking error *e(t)* asymptotically converges to zero on a sliding plane $(v=0)$ after an appropriate control time. As is easily seen from eqn. (10), asymptotic adaptation (the state of the objectives approaching the state of the model asymptotically) is possible when the variable gain h_0 is selected so as to meet the relation $h_0 > h_{0C}$ (critical gain).

3. **Experimental methods**

When the cantilever beam is modelled as a singledegree-of-freedom vibration system, the state equation is given, after identification of the system parameters, bY

$$
\dot{x} = Ax + bu \tag{11}
$$

where

$$
A = \begin{bmatrix} 0 & 1 \\ -713.1 & -0.1214 \end{bmatrix}
$$

\n
$$
b = \begin{bmatrix} 0, & 0.4961 \end{bmatrix}^{T}
$$

\n
$$
x = \begin{bmatrix} x_1, & x_2 \end{bmatrix}^{T} = [x, \dot{x}]^{T}
$$
 (12)

The reference model used is expressed by

$$
\dot{x}_{\mathbf{m}} = A_{\mathbf{m}} x_{\mathbf{m}} + b_{\mathbf{m}} u_{\mathbf{m}} \tag{13}
$$

where

$$
A_{\mathbf{m}} = \begin{bmatrix} 0 & 1 \\ -\omega_{\mathbf{n}}^2 & -\zeta \omega_{\mathbf{n}} \end{bmatrix}
$$

\n
$$
b_{\mathbf{m}} = \begin{bmatrix} 0, & \omega_{\mathbf{n}}^2 \end{bmatrix}^{\mathbf{T}}
$$

\n
$$
x_{\mathbf{m}} = \begin{bmatrix} x_{\mathbf{m}1}, & x_{\mathbf{m}2} \end{bmatrix}^{\mathbf{T}}
$$
 (14)

with the natural angular frequency being ω_n (rad s⁻¹) and the damping ratio ζ .

As the design condition for the system to be controlled, there exist the following model-matching conditions:

$$
C_{A_1} = \frac{1}{\omega_n^2} \left[-\omega_n^2 + 713.1, -2\zeta\omega + 0.12134 \right]
$$

\n
$$
C_{A_2} = 0
$$

\n
$$
C_B = \frac{0.4961}{\omega_n^2}
$$

\n
$$
C_B > 0
$$
\n(15)

Figure 2 shows the overall control system including the longitudinal-type cantilever beam system representing the control objectives. The lumped mass m_1 (1.751 kg) carrying the auxiliary mass $m₂$ (0.153 kg) at its upper part not only operates as a lumped mass in the flexural vibration but also plays the role of the actuator which generates electromagnetically the vibration control force $u(t)$ between masses m_1 and m_2 . The vibration displacement $x_1(t)$ is measured by a vibration meter (SANE1 Charge Amp). The beam is made of brass plate (300 *x* 35 *x* 3 mm3). A 16-bit PC9801 VX21 (+ i80286 (10 MHz)) microcomputer is used to implement the dynamic compensator mentioned above. In order to enhance the execution speed, the programme is written both in c language **(TURBO c 2.0)** for the main task and in assembly language for the A/D and D/A conversion. Additional enhancement of the execution speed is achieved with a coprocessor (i80287) for floating point calculations. The A/D converter (CON-TEC) has 12-bit resolution and 20 μ s conversion speed.

Fig. 2. Schematic view of the experimental system.

Fig. 3. Real-time controlled responses.

4. Results and discussion

Letting the reference model parameters be $\omega_n = 6.0$ rad s^{-1} and $\zeta = 0.8$, the vibration-controlled responses in the real system are as shown in Figs. $3(a)$ and $3(b)$, corresponding to the design parameters $Q = diag[q_1,$ q_2 = diag[10³, 1] and diag[1, 1] respectively. It is easily seen that the vibration-controlled effects become better as the relative largeness $(q_1 \gg q_2)$ becomes more noticeable. When the control input $sgn(v)$ is replaced with cont(v)=v/(||v||+ Δ) (from the viewpoint of the control) and the optimal value of Δ is taken as $\Delta_{opt} = 0.2$, the maximum damping capacity $\delta_{1\,\text{max}} = 0.3248$ (including the damping $\delta_{1f} = 0.012$ under free vibration) is obtained at $Q = diag[10^3, 1]$ as shown in Fig. 3(c). The damping capacity is estimated by the logarithmic decrement formula [2]

$$
\delta_{\mathsf{l}} = \frac{1}{n} \ln \left(\frac{h_1}{h_{n+1}} \right) \tag{16}
$$

where h_1 and h_{n+1} are the first and $(n+1)$ th decaying vibration amplitudes respectively. The maximum damping $\delta_{\text{l max}}$ amounts to about 4.5 times the maximum damping Q_m^{-1} = 0.023 observed with the ferromagnetic high damping metal SIA [6].

5. Conclusions

The active vibration-damping effect achieved by constructing a computer-controlled system has been discussed in comparison with the material damping effect observed with the ferromagnetic high damping alloy SIA.

In the Lyapunov equation related to the control law characterizing the active damping system, the greater the relative largeness of q_1 to q_2 is, the higher the active damping effect becomes. When the control input function sgn(v) is replaced with cont(v) = $v/(||v|| + \Delta)$ and the optimal value of Δ is taken as $\Delta_{opt} = 0.2$, the maximum active logarithmic damping $\delta_{1\,\text{max}} = 0.3248$ (including the damping $\delta_{1f} = 0.012$ under free vibration) is obtained at $Q = diag[q_1, q_2] = diag[10^3, 1]$. The value of $\delta_{1\,\text{max}}$ amounts to 4.5 times the maximum damping $Q_{\rm m}^{-1}$ observed with SIA.

References

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